Combinatorial Distribution

The combinatorial distribution is the individual count of a particular item z in a given column c for n items taken r at a time. The individual count can be derived by multiplying the number of factor set sub-combinations before the item z and left of the column c by the number of factor set sub-combinations after the item z and right of the column c. Below is an example of the total count for the number 4 in column 3 for 7 numbers taken 5 at a time. Each sub-combination before and left occurs exactly once with the sub-combinations after and right. The total set of sub-combinations occurring in the 5 of 7 combination set can be reduced down to a single factor set of sub-combinations be removing the duplicates.

Factor Set of Sub-combinations before number 4 and left of column 3				Column 1	Column 2	Column 3	Column 4	Column 5			Factor Set of Sub-combinations after number 4 and right of column 3	
				1	2	3	4	5				
				1	2	3	4	6				
				1	2	3	4	7				
				1	2	3	5	6				
				1	2	3	5	7				
				1	2	3	6	7				
		1	2	1	2	4	5	6	5	6		
		1	2	1	2	4	5	7	5	7		
		1	2	1	2	4	6	7	6	7		
				1	2	5	6	7				
1	2	1	3	1	3	4	5	6	5	6	5	6
1	3	1	3	1	3	4	5	7	5	7	5	7
2	3	1	3	1	3	4	6	7	6	7	6	7
				1	3	5	6	7				
				1	4	5	6	7				
		2	3	2	3	4	5	6	5	6		
		2	3	2	3	4	5	7	5	7		
		2	3	2	3	4	6	7	6	7		
				2	3	5	6	7				
				2	4	5	6	7				
				3	4	5	6	7				

Table 1 – Factor Set of Sub-combinations

The factor set of sub-combinations before and left is a combination of 3 items taken 2 at a time and the factor set of sub-combinations after and right is also 3 items taken 2 at a time. The set of items before and left is {1, 2, 3} and the set of items after and right is {5, 6, 7}. The combination function can be used to describe the counts in the combinatorial distribution and is derived from the permutation function and factorial function. The factorial is defined as follows.

(1) $$n! = n \cdot (n-1) \cdot (n-1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1 \text{ and } 0! = 1$$

The permutation function is defined as

(2) $$P(n,r) = \frac{n!}{(n-r)!} \text{ where } n \geq r \geq 0$$

The combination function is then defined as

(3) $$C(n,r) = \frac{P(n,r)}{r!} \text{ where } n \geq r \geq 0$$

The combinatorial distribution function is derived as follows. Multiply the total factor set of sub-combinations before and after the particular item z in n items taken r at a time in a particular column c. The total factor set of sub-combinations before and left is

(4) $$C(z-1, c-1)$$

The total factor set of sub-combinations after and right is

(5) $$C(n-z, r-c)$$

The combinatorial distribution is then

(6) $$D(n,r,c,z) = C(n-1,r-1) \cdot C(n-z, r-c)$$

For the example in table 1, the total number of factor set sub-combinations before and left is (4 - 1) items taken (3 – 1) at a time and the total number of factor set sub-combinations is (7 – 4) items taken (5 – 3) at a time. The work out for this example as follows.

(7) $$\begin{gathered} \text{if } n = 7, r = 5, c = 3, z = 4 \\ \text{then } D(7,5,3,4) = C(4-1, 3-1) \cdot C(7-4, 5-3) \end{gathered}$$

(8) $$D(7,5,3,4) = C(3,2) \cdot C(3,2)$$

(9) $$D(7,5,3,4) = 3 \cdot 3$$

(10) $$D(7,5,3,4) = 9$$

The combinatorial distribution function applied to the total set of combinations for 7 numbers taken 5 at a time is shown below in table 2.

7 items taken 5 at a time						
n = 7 r = 5		c				
		1	2	3	4	5
z	1	15				
	2	5	10			
	3	1	8	6		
	4		3	9	3	
	5			6	8	1
	6				10	5
	7					15

Table 2 – Combinatorial Distribution

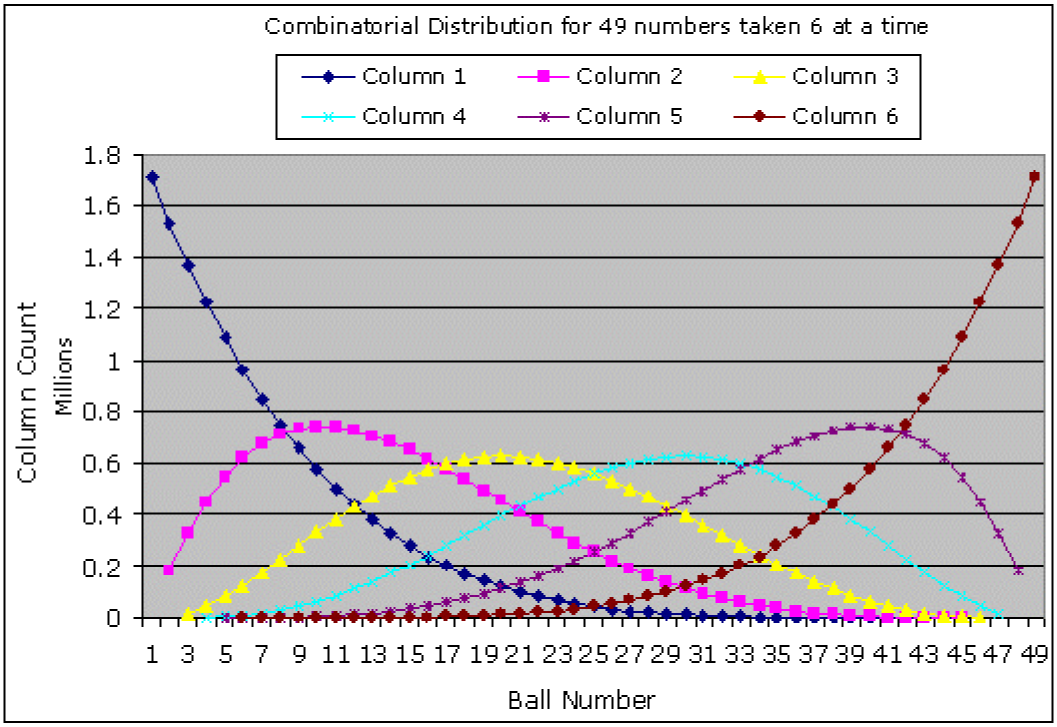
Next is an example of the combinatorial distribution applied to a 6 of 49 lottery configuration. Table 3 shows the individual counts of each number in its column.

49 items taken 6 at a time							
n = 49 r = 6		c					
		1	2	3	4	5	6
z	1	1712304					
	2	1533939	178365				
	3	1370754	326370	15180			
	4	1221759	446985	42570	990		
	5	1086008	543004	79464	3784	44	
	6	962598	617050	123410	9030	215	1
	7	850668	671580	172200	17220	630	6
	8	749398	708890	223860	28700	1435	21
	9	658008	731120	276640	43680	2800	56
	10	575757	740259	329004	62244	4914	126
	11	501942	738150	379620	84360	7980	252
	12	435897	726495	427350	109890	12210	462
	13	376992	706860	471240	138600	17820	792
	14	324632	680680	510510	170170	25025	1287
	15	278256	649264	544544	204204	34034	2002
	16	237336	613800	572880	240240	45045	3003
	17	201376	575360	595200	277760	58240	4368
	18	169911	534905	611320	316200	73780	6188
	19	142506	493290	621180	354960	91800	8568
	20	118755	451269	624834	393414	112404	11628
	21	98280	409500	622440	430920	135660	15504
	22	80730	368550	614250	466830	161595	20349

	23	65780	328900	600600	500500	190190	26334
	24	53130	290950	581900	531300	221375	33649
	25	42504	255024	558624	558624	255024	42504
	26	33649	221375	531300	581900	290950	53130
	27	26334	190190	500500	600600	328900	65780
	28	20349	161595	466830	614250	368550	80730
	29	15504	135660	430920	622440	409500	98280
	30	11628	112404	393414	624834	451269	118755
	31	8568	91800	354960	621180	493290	142506
	32	6188	73780	316200	611320	534905	169911
	33	4368	58240	277760	595200	575360	201376
	34	3003	45045	240240	572880	613800	237336
	35	2002	34034	204204	544544	649264	278256
	36	1287	25025	170170	510510	680680	324632
	37	792	17820	138600	471240	706860	376992
	38	462	12210	109890	427350	726495	435897
	39	252	7980	84360	379620	738150	501942
	40	126	4914	62244	329004	740259	575757
	41	56	2800	43680	276640	731120	658008
	42	21	1435	28700	223860	708890	749398
	43	6	630	17220	172200	671580	850668
	44	1	215	9030	123410	617050	962598
	45		44	3784	79464	543004	1086008
	46			990	42570	446985	1221759
	47				15180	326370	1370754
	48					178365	1533939
	49						1712304

Table 3 – Lottery Example for 6 of 49

The following graph shows the typical characteristic pattern for combinatorial distribution.



Graph 1 – Lottery Example for 6 of 49

From the graph, the relative count for each item in a given column shows a skewed distribution for each column and illustrates some of the symmetry between the columns. The combinatorial distribution function can be used to approximate random distributions, help define combinatorial symmetry and estimate rates of reoccurrence for random selections of a particular item in a given column as a few examples. These will be explored in different sections.